Grey Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	E	E	D	B	C	B	A	C	C	C	D	A	C	E	A	A	D	B	C	C	E	D	D	C

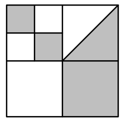
1. Which of these fractions has the largest value?

A $\frac{8+5}{3}$ B $\frac{8}{3+5}$ C $\frac{3+5}{8}$ D $\frac{8+3}{5}$ E $\frac{3}{8+5}$

Solution **A**

The values of the fractions shown are $\frac{13}{3} = 4\frac{1}{3}$, $\frac{8}{8} = 1$, $\frac{8}{8} = 1$, $\frac{11}{5} = 2\frac{1}{5}$ and $\frac{3}{13}$. Hence the fraction which has the largest value is $\frac{8+5}{3}$.

2. A large square is divided into smaller squares. In one of the smaller squares a diagonal is also drawn, as shown. What fraction of the large square is shaded?



A $\frac{4}{5}$ B $\frac{3}{8}$ C $\frac{4}{9}$ D $\frac{1}{3}$ E $\frac{1}{2}$

Solution **E**

The shaded square in the lower right corner of the large square is $\frac{1}{4}$ of the large square. The shaded triangle is half of $\frac{1}{4}$ of the large square. Hence it is $\frac{1}{8}$ of the large square. The two small shaded squares in the upper left corner together are half of $\frac{1}{4}$, or $\frac{1}{8}$, of the large square. Therefore the fraction of the large square that is shaded is $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$.

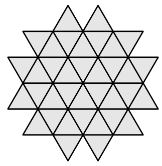
3. There are 4 teams in a football tournament. Each team plays every other team exactly once. In each match, the winner gets 3 points and the loser gets 0 points. In the case of a draw, both teams get 1 point. After all matches have been played, which of the following total number of points is it impossible for any team to have obtained?

A 4 B 5 C 6 D 7 E 8

Solution **E**

Each team plays exactly three matches. Hence the maximum number of points any team can obtain is $3 \times 3 = 9$. A draw only gets 1 point. Hence the next highest total number of points possible, from two wins and a draw, is $2 \times 3 + 1 = 7$. Therefore it is impossible to obtain 8 points.
(Note: totals of 4, 5 and 6 points can be obtained by one win, one draw and one loss, one win and two draws and two wins and a loss respectively.)

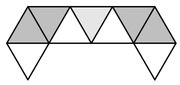
4. The diagram shows a shape made up of 36 identical small equilateral triangles. What is the smallest number of small triangles identical to these that could be added to the shape to turn it into a hexagon?



A 10 B 12 C 15 D 18 E 24

Solution **D**

To turn the figure given in the question into a hexagon by adding the smallest number of triangles, two triangles should be added to create each vertex of the hexagon (shaded dark grey) and one triangle added to create each edge (shaded light grey) as shown on the right for one edge. Since a hexagon has six vertices and six edges, the smallest number of triangles required is $6 \times (2 + 1) = 18$.



5. Kanga wants to multiply three different numbers from the following list: -5, -3, -1, 2, 4, 6. What is the smallest result she could obtain?

A -200 B -120 C -90 D -48 E -15

Solution **B**

The result furthest from zero is obtained by multiplying the three numbers furthest from zero, namely -5, 4 and 6. This gives -120, which is negative and hence is the smallest possible result.

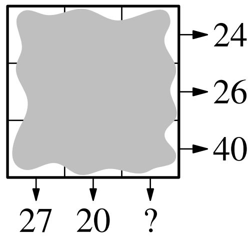
6. John always walks to and from school at the same speed. When he walks to school along the road and walks back using a short cut across the fields, he walks for 50 minutes. When he uses the short cut both ways, he walks for 30 minutes. How long does it take him when he walks along the road both ways?

A 60 minutes B 65 minutes C 70 minutes D 75 minutes E 80 minutes

Solution **C**

Since using the short cut both ways takes 30 minutes, using the short cut one way takes 15 minutes. Hence, since walking to school by road and walking back using the short cut takes 50 minutes, walking by road takes $(50 - 15)$ minutes = 35 minutes. Therefore walking by road both ways takes 2×35 minutes = 70 minutes.

7. Each cell of a 3×3 square has a number written in it. Unfortunately the numbers are not visible because they are covered in ink. However, the sum of the numbers in each row and the sum of the numbers in two of the columns are all known, as shown by the arrows on the diagram. What is the sum of the numbers in the third column?

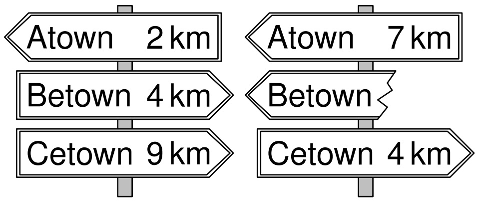


A 41 B 43 C 44 D 45 E 47

Solution **B**

The sum of the row totals is the sum of all the nine numbers in the 3×3 square. Likewise, the sum of the column totals is the sum of these nine numbers. Therefore $24+26+40 = 27+20+x$, where x is the sum of the numbers in the third column. Therefore $90 = 47 + x$ and hence $x = 43$. Therefore the sum of the numbers in the third column is 43.

8. The shortest path from Atown to Cetown runs through Betown. The two signposts shown are set up at different places along this path. What distance is written on the broken sign?



A 1 km B 3 km C 4 km D 5 km E 9 km

Solution **A**

The information on the signs pointing to Atown and the signs pointing to Cetown both tell us that the distance between the signs is $(7-2)$ km $= (9-4)$ km $= 5$ km. Therefore the distance which is written on the broken sign is $(5-4)$ km $= 1$ km.

9. Anna wants to walk 5 km on average each day in March. At bedtime on 16th March, she realises that she has walked 95 km so far. What distance does she need to walk on average for the remaining days of the month to achieve her target?

A 5.4 km B 5 km C 4 km D 3.6 km E 3.1 km

Solution **C**

The total distance Anna wants to walk is (31×5) km $= 155$ km. Since she has walked 95 km up to the 16th of March, she has $(155-95)$ km $= 60$ km to walk in $(31-16)$ days $= 15$ days. Therefore the distance, in km, that she needs to average per day is $60 \div 15 = 4$.

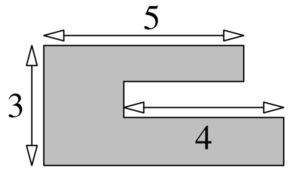
10. Every pupil in a class either swims or dances. Three fifths of the class swim and three fifths dance. Five pupils both swim and dance. How many pupils are in the class?

A 15 B 20 C 25 D 30 E 35

Solution **C**

Since three fifths of the class swim, three fifths of the class dance and no-one does neither, the fraction of the class who do both is $\frac{3}{5} + \frac{3}{5} - 1 = \frac{1}{5}$. Hence, since 5 pupils do both, the number of pupils in the class is $5 \times 5 = 25$.

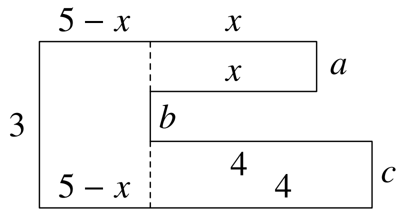
11. Sacha's garden has the shape shown. All the sides are either parallel or perpendicular to each other. Some of the dimensions are shown in the diagram. What is the length of the perimeter of Sacha's garden?



A 22 B 23 C 24 D 25 E 26

Solution **C**

Divide the garden up and let the lengths of the various sides be as shown on the diagram. Since all sides are either parallel or perpendicular, $a + b + c = 3$. Therefore the perimeter of Sacha's garden is $3 + 5 + a + x + b + 4 + c + (4 + (5 - x))$ $= 21 + a + b + c = 24$.



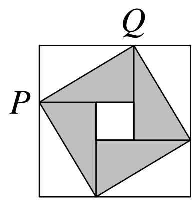
12. Werner's salary is 20% of his boss's salary. By what percentage is his boss's salary larger than Werner's salary?

A 80% B 120% C 180% D 400% E 520%

Solution **D**

Werner's salary is 20% of his boss's salary. Therefore his boss's salary is $\frac{100}{20} = 5$ times Werner's salary. Hence his boss's salary is 500% of Werner's salary and so is 400% larger.

13. The pattern on a large square tile consists of eight congruent right-angled triangles and a small square. The area of the tile is 49 cm^2 and the length of the hypotenuse PQ of one of the triangles is 5 cm. What is the area of the small square?



A 1 cm^2 B 4 cm^2 C 9 cm^2 D 16 cm^2 E 25 cm^2

Solution **A**

Since the four rectangles are congruent, the diagonal PQ is also the side of a square. This square has area $(5 \times 5)\,\text{cm}^2 = 25\,\text{cm}^2$. Therefore the total area of the rectangles outside the square with side PQ but inside the large square is $(49 - 25)\,\text{cm}^2 = 24\,\text{cm}^2$. However, this is also equal to the total area of the triangles inside the square with side PQ. Therefore the area of the small square is $(25 - 24)\,\text{cm}^2 = 1\,\text{cm}^2$.

14. Andrew buys 27 identical small cubes, each with two adjacent faces painted red. He then uses all of these cubes to build a large cube. What is the largest number of completely red faces that the large cube can have?

A 2 B 3 C 4 D 5 E 6

Solution **C**

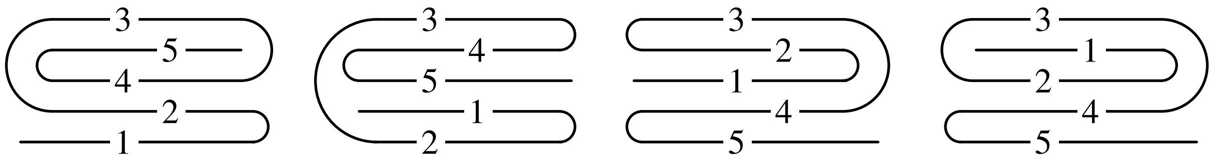
Since no small cubes have three faces painted red, it is impossible to build a large cube with three faces that meet at a vertex painted red. Therefore no more than four of the faces can be red and, without loss of generality, let us consider that they are the front, back and sides of the cube. We can construct a large cube with four faces completely red by arranging that the 12 small cubes along the four vertical edges of the large cube have two red faces showing and the 12 small cubes down the centre of the four vertical faces have one red face showing. Hence it is possible to build a large cube with four completely red faces.

15. Aisha has a strip of paper with the numbers 1, 2, 3, 4 and 5 written in five cells as shown. She folds the strip so that the cells overlap, forming 5 layers. Which of the following configurations, from top layer to bottom layer, is it not possible to obtain?

1	2	3	4	5

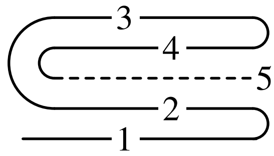
A 3, 5, 4, 2, 1 B 3, 4, 5, 1, 2 C 3, 2, 1, 4, 5 D 3, 1, 2, 4, 5 E 3, 4, 2, 1, 5

Solution **E**



A B C D

The four figures (A) to (D) give a side-view of how the strip could be folded to give the arrangements of numbers in options A to D. Figure (E) shows that it is not possible to get option E since number 5 would end up between number 4 and number 2 (as indicated by the dashed line labelled 5) rather than below number 1 as is required.



E

16. Twelve coloured cubes are arranged in a row. There are 3 blue cubes, 2 yellow cubes, 3 red cubes and 4 green cubes but not in that order. There is a yellow cube at one end and a red cube at the other end. The red cubes are all together within the row. The green cubes are also all together within the row. The tenth cube from the left is blue. What colour is the cube sixth from the left?

A green B yellow C blue D red E red or blue

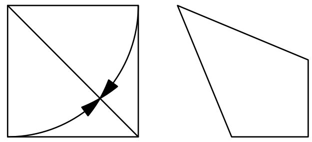
Solution **A**

We are told there is a red cube at one end and that the three red cubes are all together within the row. Therefore there is a block of three red cubes at one end. If they were at the right-hand end, the tenth cube from the left would be red. But the tenth cube from the left is blue. Hence the red cubes are at the left-hand end and a yellow cube is at the right-hand end, with a blue cube in the tenth place from the left, as shown in the diagram.

R	R	R							B		Y

The four green cubes are all together within the row and hence are somewhere between the 4th and 9th positions from the left. Whether they start at position 4 or 5 or 6 from the left, the cube in the 6th position from the left is green.

17. Bella took a square piece of paper and folded two of its sides to lie along the diagonal, as shown, to obtain a quadrilateral. What is the largest size of an angle in that quadrilateral?



A 112.5° B 120° C 125° D 135° E 150°

Solution **A**

Since the quadrilateral is formed by folding the 45° angles above and below the diagonal of the square in half, the size of the small angle of the quadrilateral is $2 \times (\frac{1}{2} \times 45°) = 45°$. One angle of the quadrilateral is 90° and the other two are equal from the construction. Therefore, since the sum of the angles in a quadrilateral is 360°, the size of the equal angles is $(360 - 90 - 45)° \div 2 = 225° \div 2 = 112.5°$.

18. How many four-digit numbers N are there, such that half of the number N is divisible by 2, a third of N is divisible by 3 and a fifth of N is divisible by 5?

A 1 B 7 C 9 D 10 E 11

Solution **D**

The information in the question tells us that N is divisible by $2 \times 2 = 4$, $3 \times 3 = 9$ and $5 \times 5 = 25$. Since these three values have no factors in common, N is divisible by $4 \times 9 \times 25 = 900$. Therefore N is a four-digit multiple of 900. The smallest such multiple is $2 \times 900 = 1800$ and the largest is $11 \times 900 = 9900$. Therefore there are 10 such four-digit numbers.

19. In the final of a dancing competition, each of the three members of the jury gives each of the five competitors 0 points, 1 point, 2 points, 3 points or 4 points. No two competitors get the same mark from any individual judge. Adam knows all the sums of the marks and a few single marks, as shown. How many points does Adam get from judge III?

	Adam	Berta	Clara	David	Emil
I	2	0			
II		2	0		
III					
Sum	7	5	3	4	11

A 0 B 1 C 2 D 3 E 4

Solution **B**

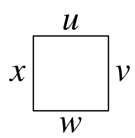
The table can be partially completed as follows. Berta scored 5 points in total. Therefore her score from judge III is 3. Clara's total is 3. Therefore she cannot have been given 4 by any of the judges. David's total is 4. He cannot have been given a score of 4 by any judge, since, if so, both the other two judges must have given him 0. This is impossible, as judges I and II give 0 to Berta and Clara, respectively. Therefore judge I gives 4 to Emil, and judge II gives 4 to either Adam or Emil. Emil's total is 11. So if he gets 4 from judges I and II, he gets 3 from judge III which is not possible as judge III gave Berta 3. Hence judge II gives 4 to Adam. Because Adam's total is 7, it now follows that Adam gets a score of 1 from judge III.
(Note: the final four scores are not uniquely determined.)

	Adam	Berta	Clara	David	Emil
I	2	0			④
II	④	2	0	①	③
III	①	③			④
Sum	7	5	3	4	11

20. Harriet writes a positive integer on each edge of a square. She also writes at each vertex the product of the integers on the two edges that meet at that vertex. The sum of the integers at the vertices is 15. What is the sum of the integers on the edges of the square?

A 6 B 7 C 8 D 10 E 15

Solution **C**

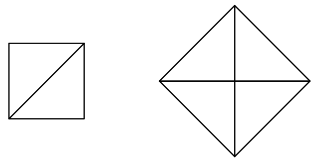


Let the integers written on the edges of the square be u, v, w and x as shown in the diagram. Therefore, since the integer written at each vertex is the product of the integers written on the edges that meet at that vertex, the integers at the vertices are ux, uv, vw and wx. Therefore $ux + uv + vw + wx = 15$ and hence $(u + w)(v + x) = 15$. Since u, v, w and x are positive integers, the smallest either of $u + w$ or $v + x$ can be is $1 + 1 = 2$. Therefore the only possible multiplications which give an answer of 15 are $3 \times 5 = 15$ and $5 \times 3 = 15$. In each case, the sum of the integers on the edges of the square is $3 + 5 = 8$.

21. Sophia has 52 identical isosceles right-angled triangles. She wants to make a square using some of them. How many different-sized squares could she make?

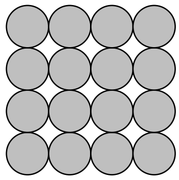
A 6 B 7 C 8 D 9 E 10

Solution **C**



Sophia can make a small square by joining two of her isosceles right-angled triangles, as shown in the first diagram. She can then join 4 or 9 or 16 or 25 of these small squares to make larger squares using up to 50 triangles. She can also create a different small square by joining four of her isosceles triangles, as shown in the second diagram. Note that the side-length of this square is $\sqrt{2}$ times the side-length of the previous small square. She can then join 4 or 9 of this second type of small square to make larger squares using up to 36 triangles. The eight squares described so far all have different side-lengths and it can be shown that there are no other possible sizes of squares. Hence Sophia can make 8 different-sized squares with the triangles she has.

22. Cleo builds a pyramid with identical metal spheres. Its square base is a 4×4 array of spheres, as shown in the diagram. The upper layers are a 3×3 array of spheres, a 2×2 array of spheres and a single sphere at the top. At each point of contact between two spheres, a blob of glue is placed. How many blobs of glue will Cleo place?



A 72 B 85 C 88 D 92 E 96

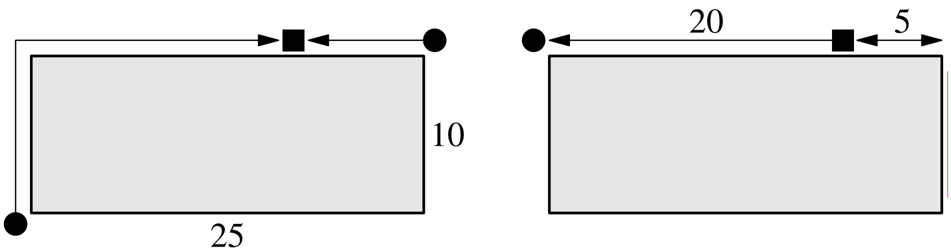
Solution **E**

Consider first how to join each individual layer. In the 4×4 layer there are $4 \times 3 \times 2 = 24$ points of contact. Similarly, in the 3×3 layer there are $3 \times 2 \times 2 = 12$ points of contact and in the 2×2 layer there are $2 \times 1 \times 2 = 4$ points of contact. Now consider how to join the layers together. To join the 4×4 layer to the 3×3 layer, each sphere in the 3×3 layer has 4 points of contact with the lower layer, making $3 \times 3 \times 4 = 36$ points of contact. Similarly to join the 3×3 layer to the 2×2 layer, each sphere in the 2×2 layer has 4 points of contact with the lower layer making $2 \times 2 \times 4 = 16$ points of contact and the single sphere in the top layer has 4 points of contact with the 2×2 layer. Hence the total number of points of contact is $24 + 12 + 4 + 36 + 16 + 4 = 96$ and therefore 96 blobs of glue are used.

23. Four children are in the four corners of a 10 m $\times$ 25 m pool. Their coach is standing somewhere on one side of the pool. When he calls them, three children get out and walk as short a distance as possible round the pool to meet him. They walk 50 m in total. What is the shortest distance the coach needs to walk to get to the fourth child's corner?

A 10 m B 12 m C 15 m D 20 m E 25 m

Solution **D**



Consider two children in opposite corners of the pool. Wherever their trainer stands, the total distance these two pupils would need to walk to meet him is half the perimeter of the pool, as illustrated in the first diagram. Therefore, if all four children walked to meet their trainer, the total distance they would walk is $(2 \times 10 + 2 \times 25)$ m $= 70$ m. Since we are given that three of the children walked 50 m in total, the distance the trainer would have to walk to get to the fourth child is $(70 - 50)$ m $= 20$ m. The second diagram shows one possible position of the trainer and child which satisfies this situation although there are others.

24. Anne, Bronwyn and Carl ran a race. They started at the same time, and their speeds were constant. When Anne finished, Bronwyn had 15 m to run and Carl had 35 m to run. When Bronwyn finished, Carl had 22 m to run. What was the length of the race?

A 135 m B 140 m C 150 m D 165 m E 175 m

Solution **D**

First note that Carl ran $(35 - 22)\,\text{m} = 13\,\text{m}$ while Bronwen ran 15 m. Let the length of the race be x m. Since their speeds were constant, the ratio of the distances they ran in any time is also constant. Therefore, since Carl ran $(x - 35)$ m while Bronwen ran $(x - 15)$ m, we have $\frac{x-35}{x-15} = \frac{13}{15}$. Therefore $15x - 15 \times 35 = 13x - 13 \times 15$ and hence $2x = 15 \times 35 - 13 \times 15$. Therefore $2x = 22 \times 15$ and hence $x = 11 \times 15$. Therefore the distance they ran is 165 m.

25.

4 1 3 2	Two digits are correct but in the wrong places.
9 8 2 6	One digit is correct and in the right place.
5 0 7 9	Two digits are correct with one of them being in the right place and the other one in the wrong place.
2 7 4 1	One digit is correct but in the wrong place.
7 6 4 2	None of the digits is correct.

The statements above give clues to the identity of a four-digit number.

What is the last digit of the four-digit number?

A 0 B 1 C 3 D 5 E 9

Solution **C**

Let's call the four-digit number N. The last clue tells us that none of the digits 7, 6, 4 or 2 is a digit in N. Then the fourth clue shows that 1 is a digit in N, but it is not the fourth digit. The first clue now tells us that N involves a 3 but not as its third digit. It also shows that 1 is not the second digit. The second clue now tells us that either 8 is the second digit of N and 9 is not one of its digits or else 9 is the first digit of N and 8 is not one of its digits. Suppose that 8 were the correct second digit. Then the third clue would tell us that both 0 and 5 were correct digits. But this would mean that all of 1, 3, 8, 0 and 5 were digits of the four-digit number N. Therefore 8 is incorrect and so 9 is correct and is the first digit. Knowing this, the third clue shows us that exactly one of 5 and 0 is correct and, moreover, it is in the right place. It can't be 5 because the first digit of N is 9. So 0 is the correct second digit. We already know 3 is correct, but is not the third digit; so the last digit of N is 3 and N is 9013.

